Average ER gain from Sacrificial Weapons

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1 Introduction

The sacrificial series is a 4-star weapon line whose skill is to allow a character's skill to proc a second time after the first with some probability. These weapons have become important for some characters who need to funnel energy into either their teammates or themselves. These units include (but are not limited to) Xingqiu and Diona. However, we still do not know exactly how much more energy these units generate for themselves. This study aims to show exactly how much more energy is generated by units using sacrificial weapons by deriving a closed form solution for the average energy gain from these weapons. Additionally, the study will reveal how refinements improve the efficacy of the sacrificial weapons.

2 Method

To simplify calculations, we assume that the energy particles generated from the skills funnel back to a unit of the corresponding element. Second, when the sacrificial weapon passive is available, the unit will always consume it immediately. This justification is reasonable since waiting to trigger a sacrificial weapon decreases the amount of energy gained per second. Third, we assume that a unit's skill generates the same amount of particles. Finally, we assume that the unit's skill is used the moment it is off cooldown.

We make several definitions below:
C_S = Cooldown of sacrificial weapon trigger
C_U = Cooldown of unit skill trigger
p_S = Probability of sacrificial weapon skill triggering
E = Energy recharged by unit's skill
E_S = Energy recharged by a unit holding a sacrificial sword
E_{gain} = Energy recharge gain overall

To calculate the average energy generated, we can examine a unit's ability to generate energy per second. This simplifies calculations that will make sense later. There are two primary cases where the cooldown of sacrificial weapons interact with the cooldown of skills. The first case is when the cooldown of the weapon skill is less than the skill (for example, Xingqiu (21s) with R5 sacrificial sword (16s)), and the second is when the cooldown of the weapon skill is more than the unit's skill (Xingqiu (21s) with R1 sacrificial sword (30s)). We first examine the former case.

2.1 When $C_S \leq C_U$

In the case when $C_S \leq C_U$, we are guaranteed to have one sacrificial weapon trigger each time we use the skill. Let an interval I be the time between the skill's cooldown, then $I = C_U$. The number of times the skill can be cast is $k = \lfloor \frac{C_S}{C_U} \rfloor$. So the baseline energy recharge per second is $\frac{kE}{I}$. By law of expectation:

$$E_S = \frac{kE + p_S E}{I} \tag{1}$$

So the percentage increase in energy recharged is $E_{gain} = \frac{E_S}{E} = \frac{\frac{kE+p_S E}{I}}{\frac{kE}{I}} = \frac{k+p_S}{k}$. This formula shows why units with short skill cooldowns such as Bennett are not as effective with sacrificial sword. As k gets large, the percentage energy gain becomes smaller.

2.2 When $C_S > C_U$

There are more issues when trying to find the expected average energy recharge when $C_S > C_U$. We are not guaranteed to get a chance to proc the sacrificial weapon passive every time we activate our skill.

We can think of sacrificial weapon procs as a binomial trial. In the event where the sacrificial weapon skill does not proc, then the skill generated E_S energy . When the sacrificial weapon procs, we will calculate the energy generated from the skill that triggered the sacrificial weapon skill and all skill triggers that happen when the sacrificial weapon is on cooldown. For example, if Xingqiu skill (21s) procs an R1 sac sword (30s), we calculate the energy generated by 3 Xingqiu E's. We can get the total energy generated and divide it by the total time taken.

Define a random variable $X = \text{Binomial}(n = n, p = p_s)$ and $q_s = 1 - p_s$. Suppose that a sac weapon procs when X = 1 and does not proc when X = 0. The energy generated when the sac weapon does not proc is E. The energy generated when the sac weapon does proc is $kE + E = (k+1)E$ where k is the number of times a unit can use their skill before the sacrificial weapon passive recharges, or $k = \lfloor \frac{C_S}{C_U} \rfloor$. Then we can define E_S as follows:

$$E_S = \frac{(n-X)(E) + X((k+1)E)}{(n-X)C_U + XkC_U}$$

The idea of the definition above is to take the total energy generated and divide it by how much time has elapsed. The first term in the denominator and the numerator represents the total energy generated from non-sac weapon triggered skills and the cooldown time of those skills respectively. The second term on both sides represents the total energy generated and the time elapsed from skills that triggered the sac sword passive.

So by linearity of expectation:

$$E_S = \frac{(n-X)(E) + X(k+1)(E)}{(n-X)C_U + XkC_U} \tag{2}$$

$$Exp[E_S] = Exp[\frac{(n-X)(E) + X((k+1)(E)}{(n-X)C_U + XkC_U}] \tag{3}$$

$$Exp[E_S] = \frac{Exp[(n-X)](E) + Exp[X](k+1)(E)}{Exp[(n-X)]C_U + Exp[X]kC_U} \tag{4}$$

$$Exp[E_S] = \frac{nq(E) + np(k+1)(E)}{nqC_U + npkC_U} \tag{5}$$

$$Exp[E_S] = \frac{q(E) + p(k+1)(E)}{qC_U + pkC_U} \tag{6}$$

$$Exp[E_S] = \frac{E(q + p(k+1))}{C_U(q+pk)} \tag{7}$$

The expected energy gain per second for a unit without a sacrificial weapon is $\frac{E}{C_U}$. So the expected energy gain per second is: The percentage gain in energy generated per second is:

$$Exp[\%E_{gain}] = \frac{\frac{q(E)+p(k+1)(E)}{qC_U+pkC_U}}{\frac{E}{C_U}} \tag{8}$$

$$Exp[\%E_{gain}] = \frac{q + p(k+1)}{q+pk} \tag{9}$$

$$Exp[\%E_{gain}] = \frac{q + p(\lfloor \frac{C_S}{C_U} \rfloor + 1)}{q + p\lfloor \frac{C_S}{C_U} \rfloor} \tag{10}$$

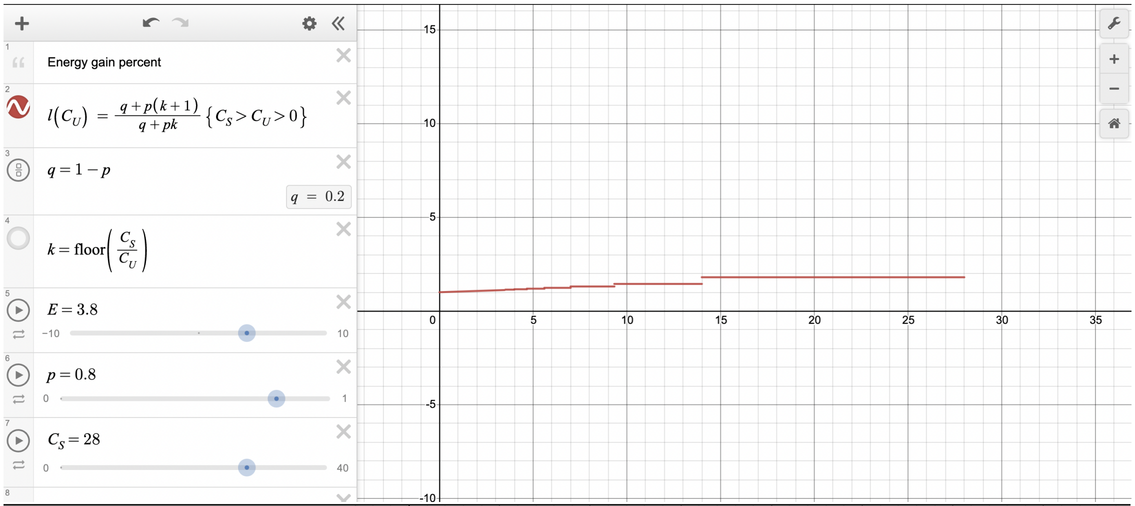
We can see that as C_U increases, the expected gain in energy recharge decreases. More rigorously:

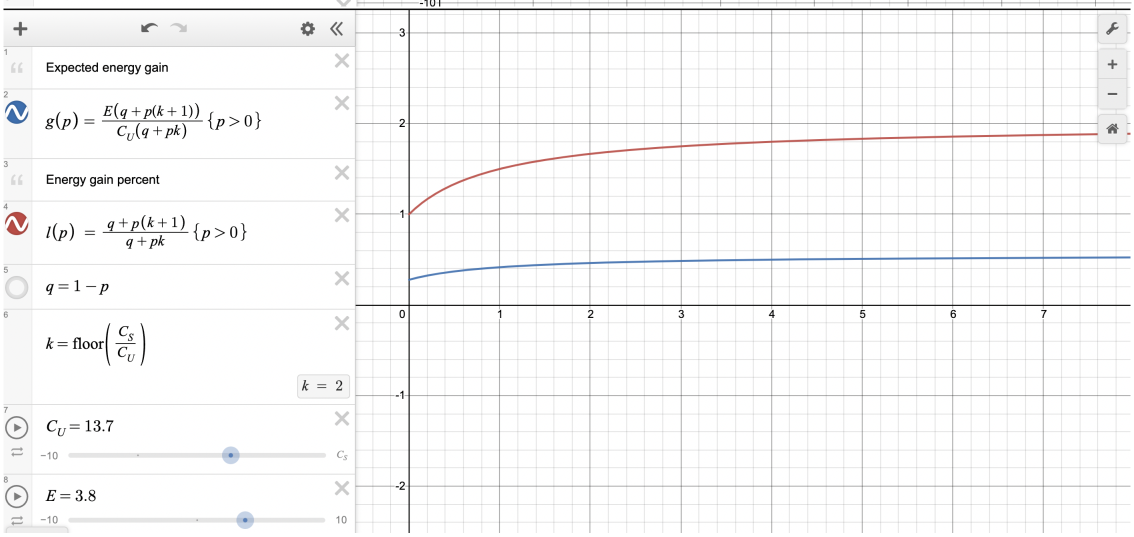
$$\lim_{C_U \to 0} \frac{q + p(\lfloor \frac{C_S}{C_U} \rfloor + 1)}{q + p\lfloor \frac{C_S}{C_U} \rfloor} = \lim_{C_U \to 0} Exp[\%E_{gain}] = 1$$

When $C_S = C_U$, you can also show that $Exp[\%E_{gain}] = 1 + p_s$. This is left as an exercise for the reader.

3 Analysis

Below is a graph of the expected gains from using a sacrificial weapon as a function of p and as a function of C_U:





We can see that the returns on increasing refinement rank decreases the higher the refinement. However, people should still refine their sacrificial swords. Using the sliders, it can also be seen that increasing k and C_U decreases the net gain. Additionally, we see that as C_U increases, the efficacy of sacrificial weapons increase.

These results show us that the best users of sacrificial sword must generate a lot of energy with one skill and also have a long cooldown time. These units happen to be Xingqiu and Diona, who both have good energy generation and relatively long skill cooldowns.